

# PROPORTION

PLANAR GEOMETRIC TRANSFORMATIONS  
AND GEOMETRIC RELATIONSHIP

The relationship regarding dimensions between two or more figures, or between two parts or a part and the whole is called PROPORTION.

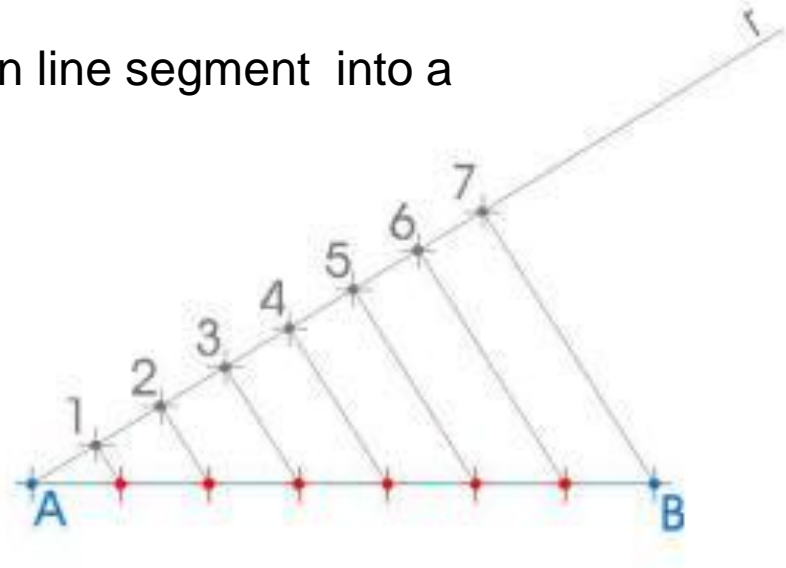
**The Thales Theorem** and **the Golden Ratio** are proportion relationships between segments

Among figures, there are some kinds of proportion relationships:

- **EQUALITY** : an equal figure can be constructed by TRANSLATION, ROTATION, TRIANGULATION, RADIATION, COORDINATES.
- **SYMMETRY**: figures have the same shape and size but they are opposite each other : REFLECTION SYMMETRY and POINT REFLECTION SYMMETRY
- **SIMILARITY**: figures have the same shape but different size.

# THALES THEOREM

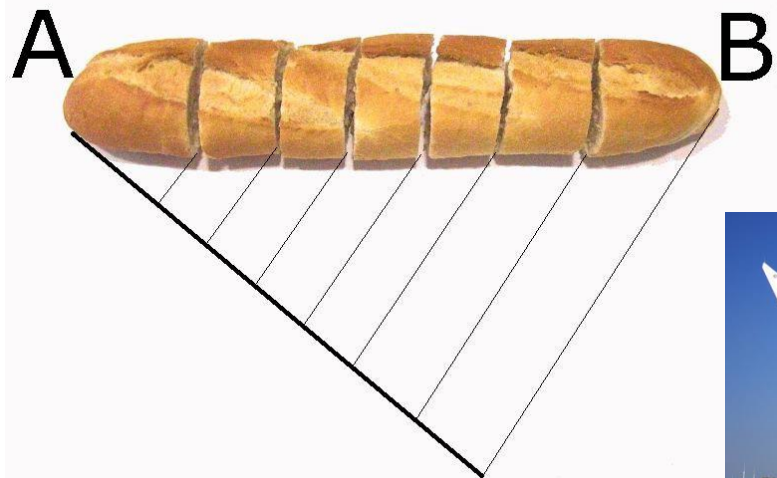
We use Thales Theorem to divide a given line segment into a number of equal parts



Using Thales we can divide a segment in equal parts.

STEPS:

1. Draw the given segment AB. This is the segment that we want to divide.
2. From point A draw an oblique ray (r).
3. Chose a measure with your compass and from point A draw on the oblique ray as many arcs (x) as parts you need.
4. Join the last point of the oblique ray with point B.
5. Draw parallels using your set square to the segment Bx from the other points on the ray.

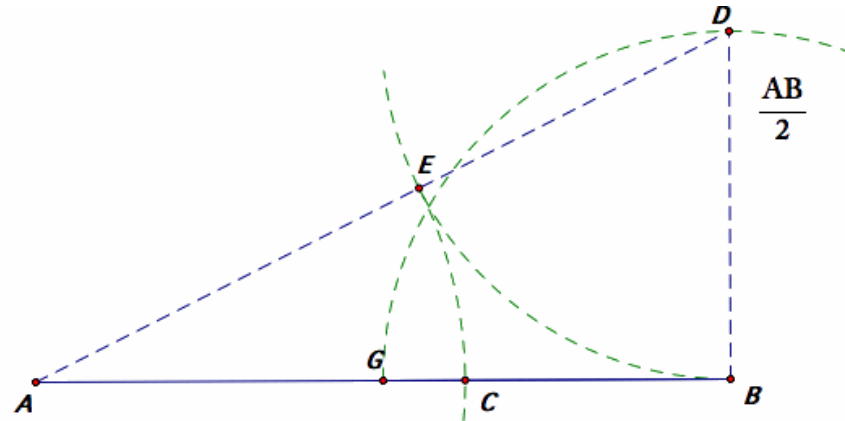


# GOLDEN RATIO

The Golden Section or Ratio is a ratio or proportion defined by the number [Phi](#) (= 1.618033988749895...)

It can be derived with a number of [geometric constructions](#), each of which divides a line segment at the unique point where: the ratio of the whole line (A) to the large segment (B) is the same as the ratio of the large segment (B) to the small segment (C).

In other words, A is to B as B is to C.

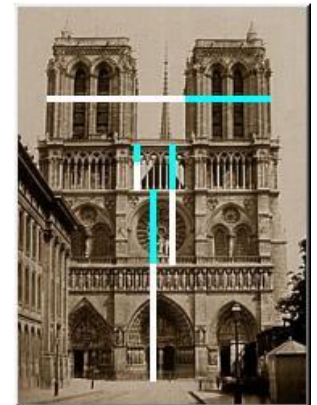
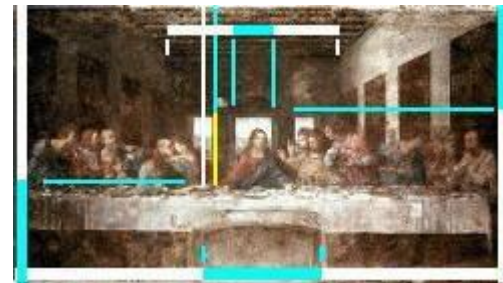


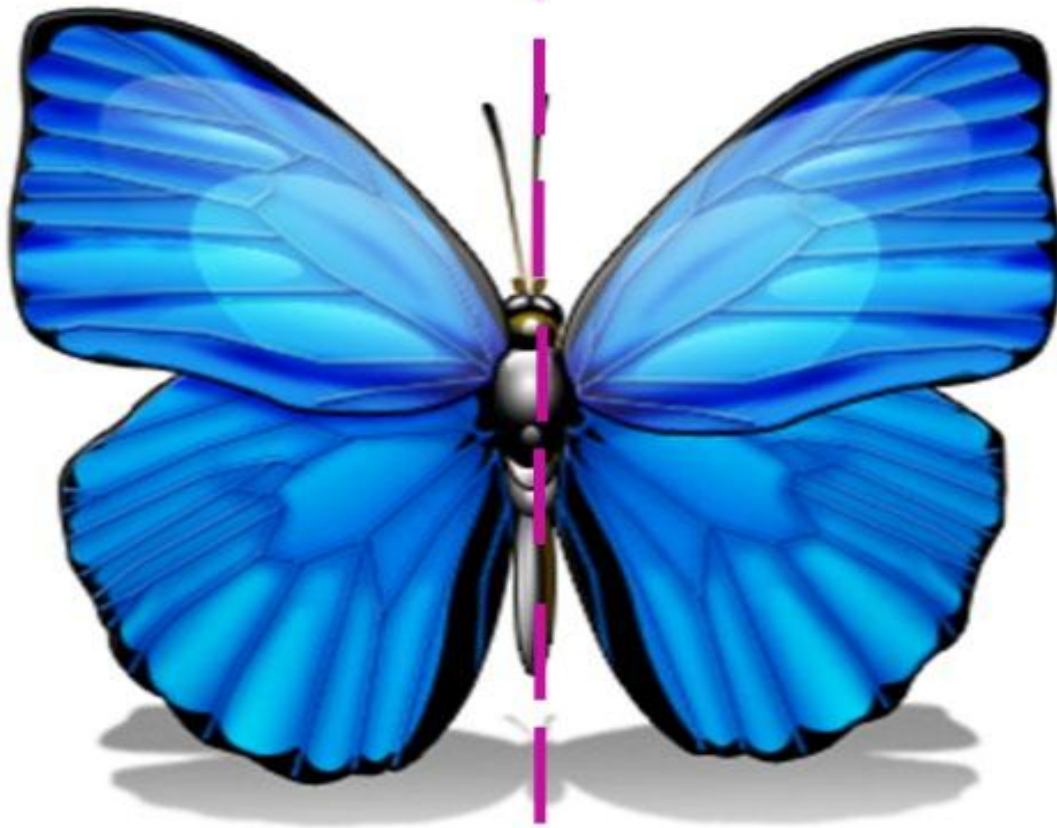
Find the midpoint G of the segment AB.

Trace a perpendicular from point B. With center at B and radius BG trace an arc which cuts the perpendicular at point D. Join D and A.

From D and radius DB trace an arc that cuts AD at point E. With center at A and radius AE trace an arc that cuts segment AB at point C. This point divides the segment AB so that AC is its golden ratio.

**This ratio has been used by mankind for centuries, for beauty and balance in the design of art and architecture**



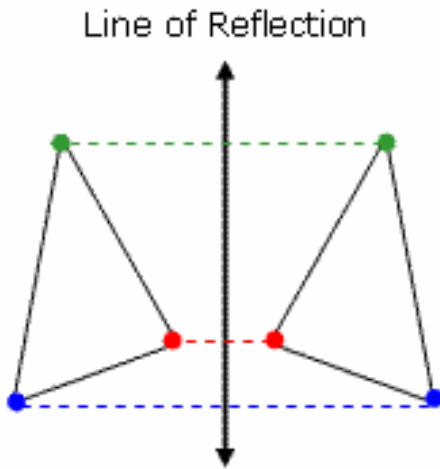


# Symmetry

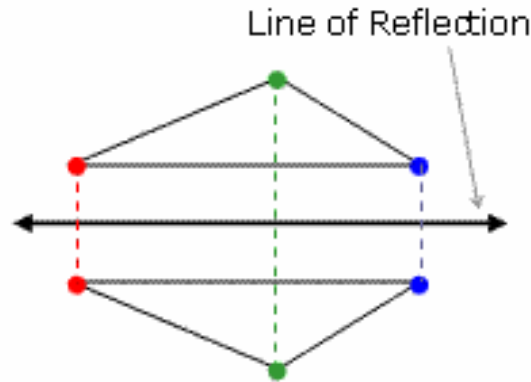
It is a geometric transformation in which every point and its symmetric are on the opposite side of an axis or a center and at the same distance from it.

# TYPES OF SYMMETRY

**REFLECTION SYMMETRY** ( axis–line of reflection): Sometimes called *Line Symmetry* or *Mirror Symmetry*. The symmetric points are over perpendicular lines to the symmetry axis, at the same distance from it and on opposite sides of it.



Horizontal Reflection  
(flips across)

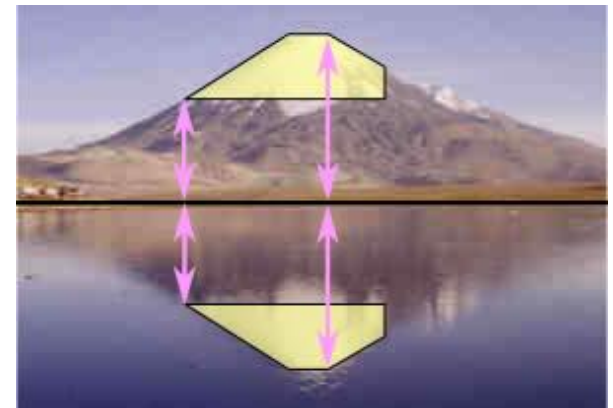


Vertical Reflection  
(flips up/down)

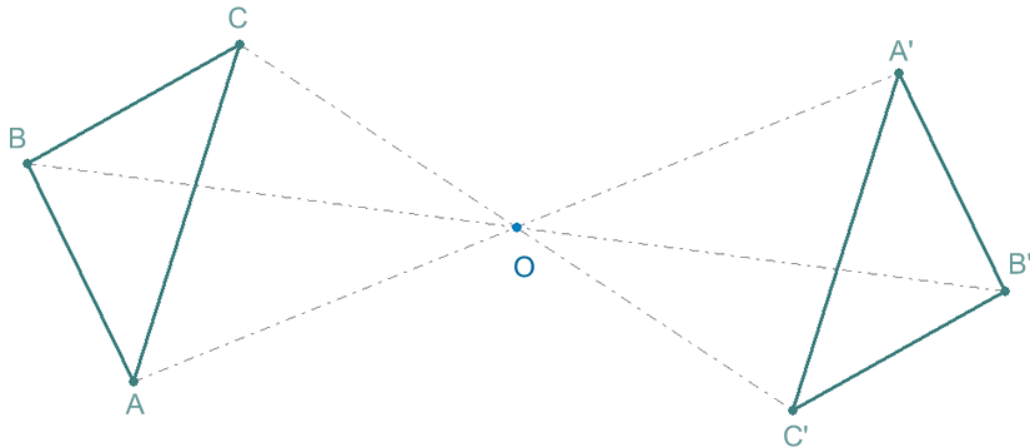
Trace the perpendicular lines to the symmetry axis through each vertex. Copy with the compass the distance from every vertex to the axis on the other side to obtain the symmetric vertex. Join the symmetric vertices.



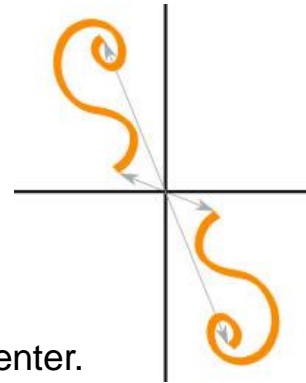
Line of Point Symmetry



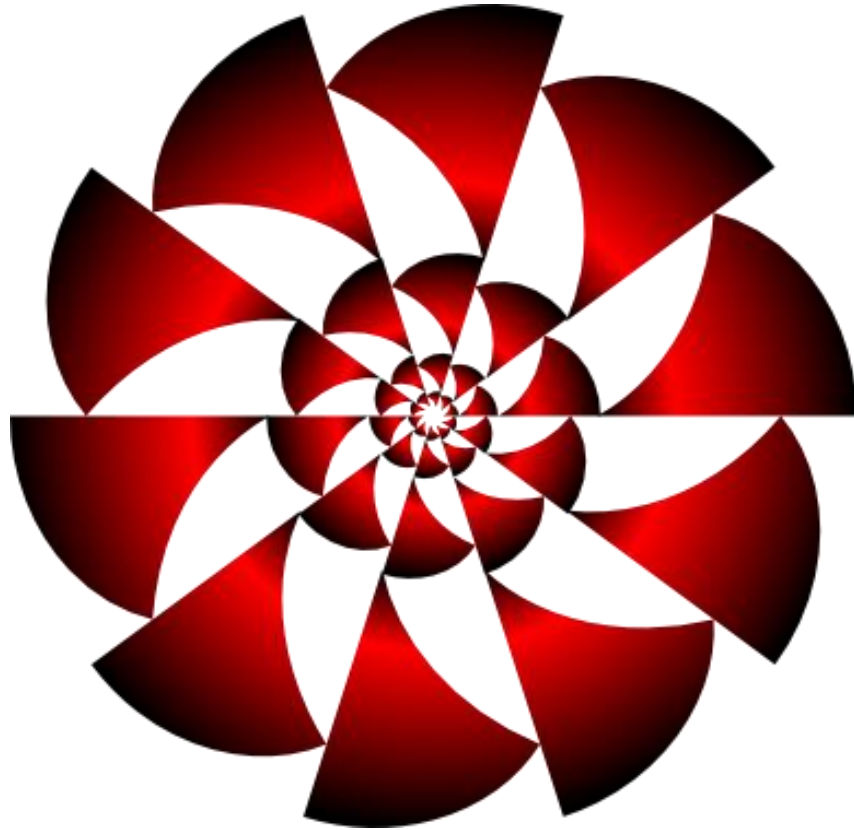
**POINT REFLECTION SIMMETRY:** ( center-point): *Also called Central symmetry.* The symmetric points are aligned with the center of symmetry  $O$ , at the same distance and on the opposite side of it.



From each vertex trace a line that goes through the symmetry center. Take the distances with the compass over the symmetry center and copy them on the other side to obtain the symmetric vertices. Join the symmetric vertices.

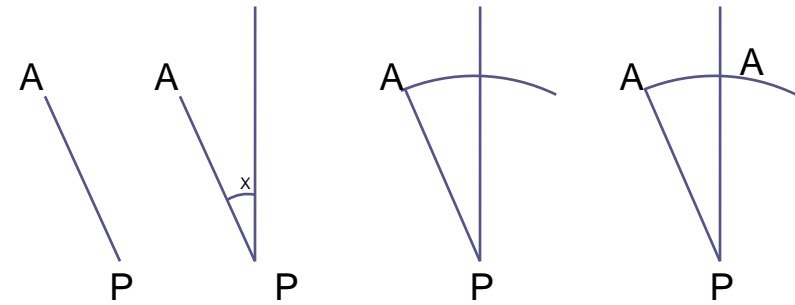




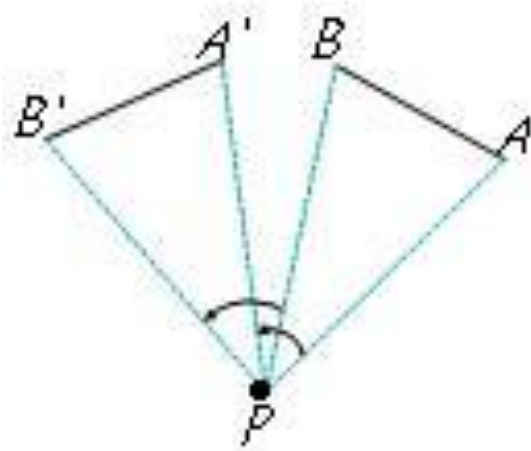


# Rotation

It is a geometric transformation in which there are a center, a rotation angle and a direction. The rotated shape or image still looks the same.

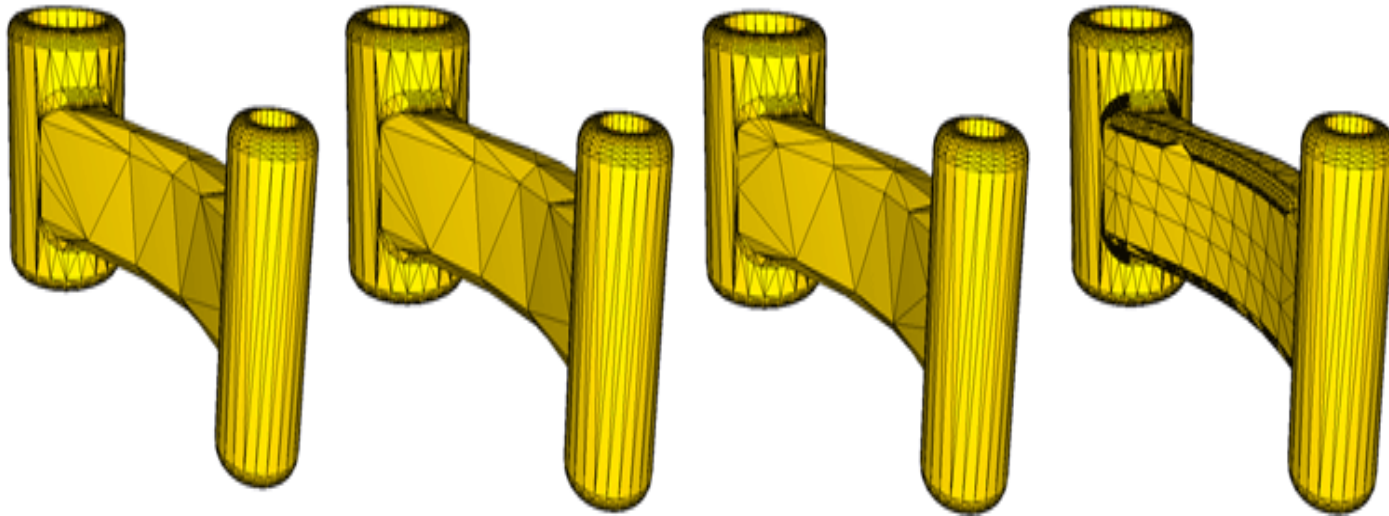


To rotate a point A "X" degrees around a center (P):  
 Trace de segment AP and with vertex on P and "X" angle  
 trace another segment forming the required "X" angle.  
 With center at P and radius PA trace an arc that cuts the  
 last segment at A .



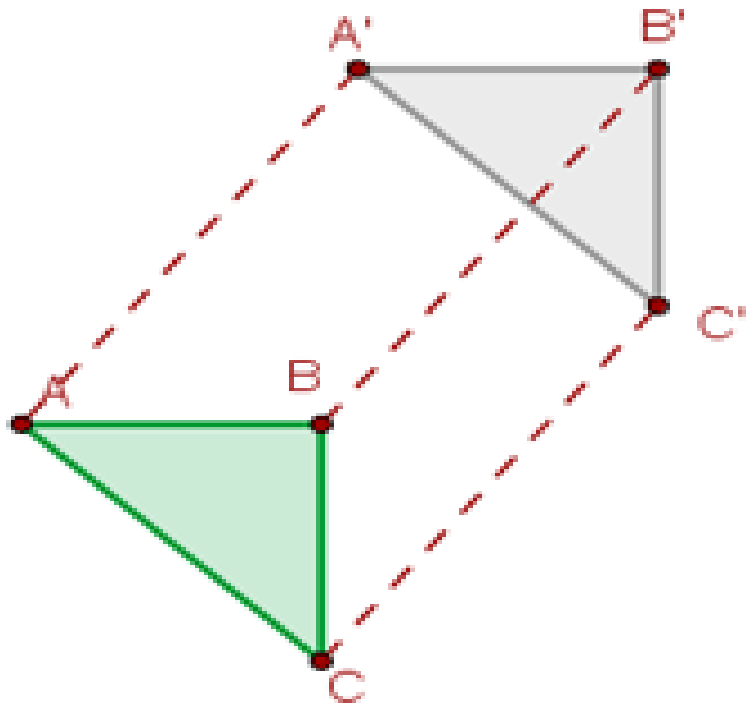
To rotate a segment, use the method above rotating point B  
 the same way.  
 Join A and B .



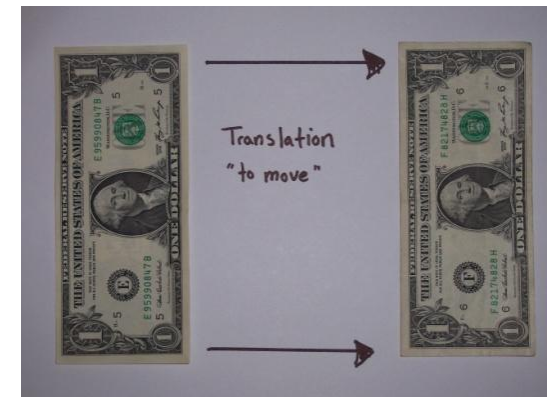


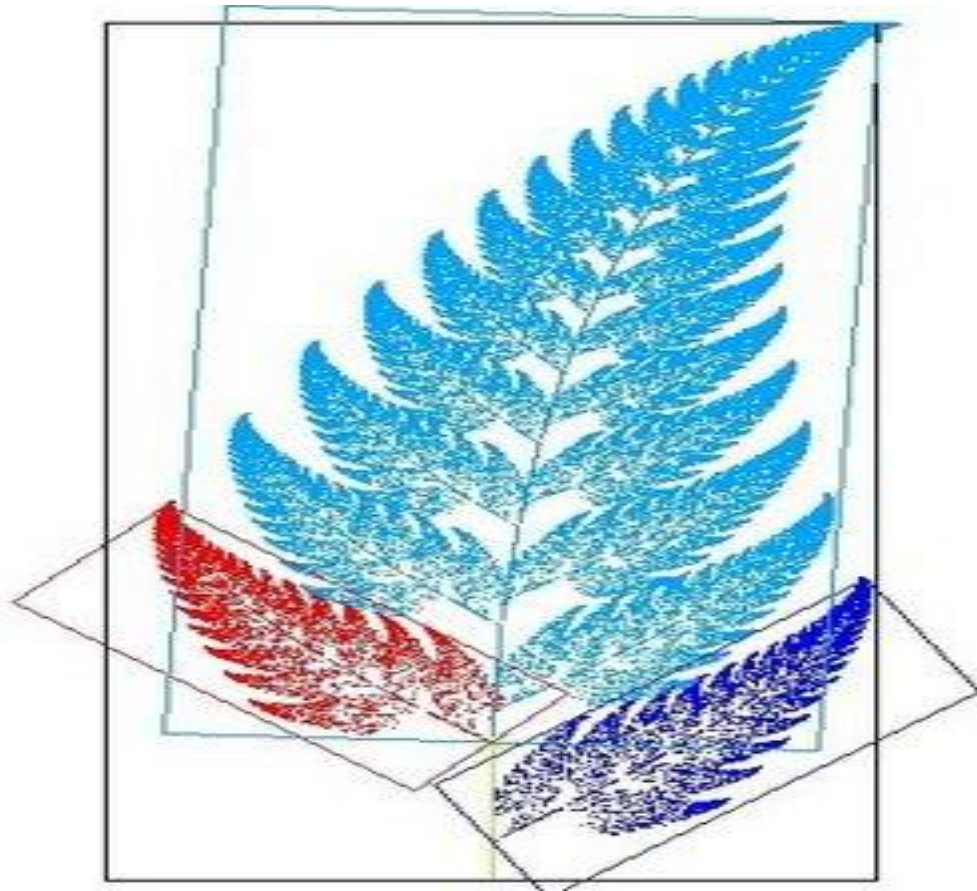
# Translation

It is a geometric transformation determined by a translation vector. This vector has a length, slope and direction



A translation motion is defined by a figure and a translation vector. It is as simple as tracing parallels to the vector slope and in the given direction by the arrow from the figure's vertices, copying the length with the compass, to obtain the translated figure.



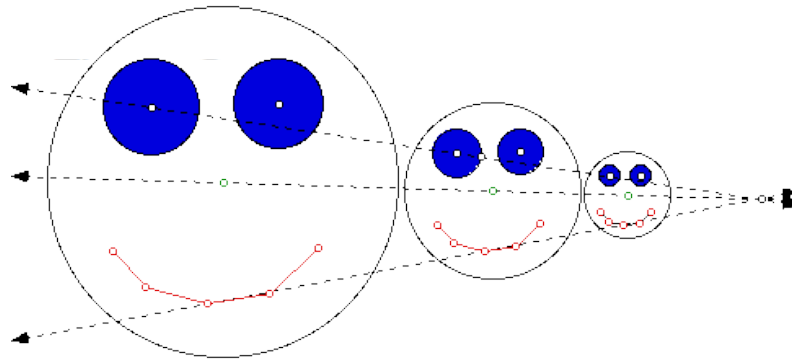
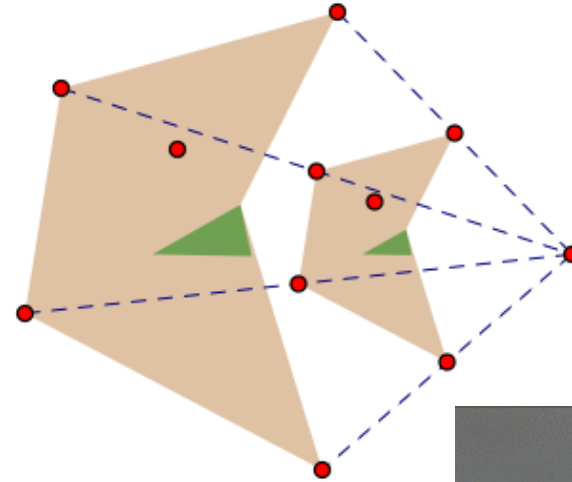
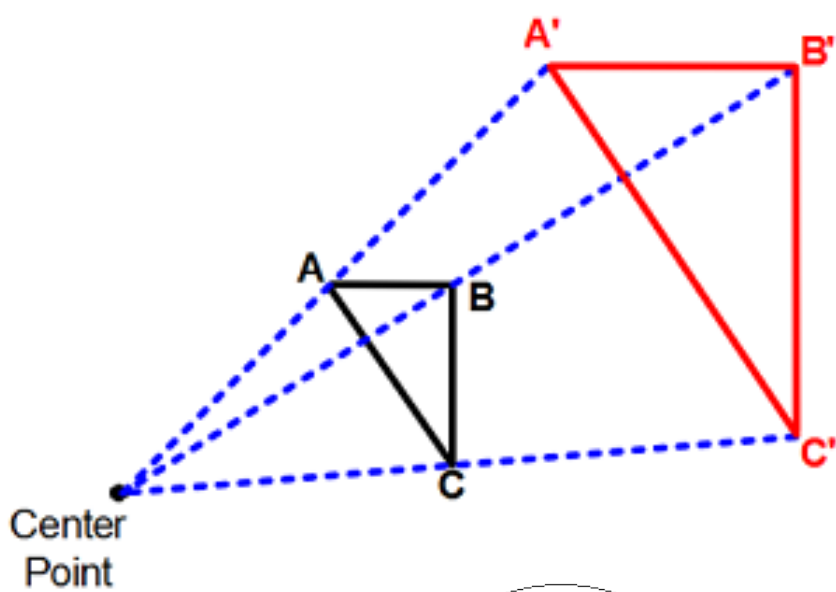


# Similarity

Two figures are similar when they have the same shape but different size and so different areas.

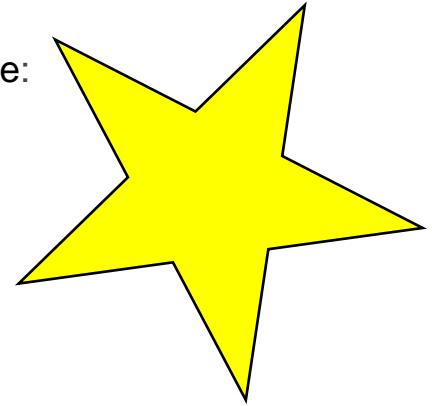
Two similar figures show the same shape, their sides are proportional and their angles are equal, but they have different sizes and areas.

To obtain a dilated figure, choose an outer point or **center point** and trace rays which go through the original figure's vertices. Extend those rays and place the points of the new figure over parallel segments to the original ones.

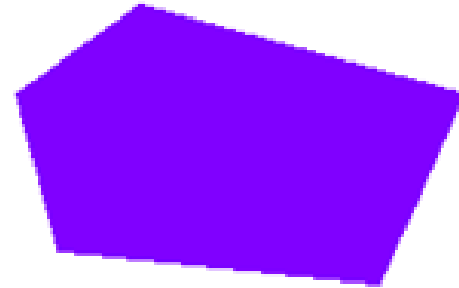
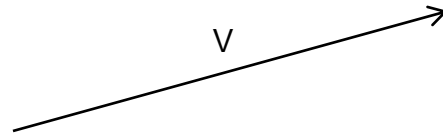


## EXERCISES

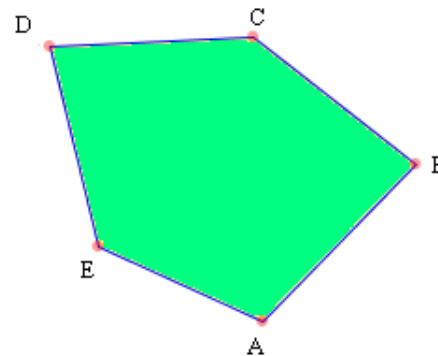
1- REFLECTION SYMMETRY and POINT REFLECTION SIMMETRY of the figure:



2-TRANSLATION of the figure given the vector "V" :



3- DILATION of the figure from an outer point:



## 4- ROTATION

### PAPER WINDMILL

#### Materials Required:

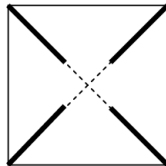
- Paper or thin card, approx. A4 size, (however if you're feeling adventurous you can make much bigger windmills)
- Paper fastener
- Doweling or thin stick, approx. 30cm, (12")
- Tape or glue
- Darning needle to make holes for paper fastener
- Ribbons for decoration, (optional)

#### Instructions:

1. Cut the paper/card into a square, (suggested size 21cm, (8") square)
2. Fold the square corner to corner in half diagonally, then open out
3. Repeat with the other diagonal corners, and open out again.



4. Colour in or decorate both sides of the paper/card at this stage if wished.
5. Cut just over halfway along each folded line, towards the centre, (do not cut all the way to the centre, or the windmill will fall apart).



6. To make the next bit easier, mark the four corners and the centre as shown in the diagram on the next page, and make a hole at each of these points, (not too close to the edge), to fit the paper-fastener through when assembling. (Remember the hole has to be large enough for the paper-

